- $[\underline{x}_{\sigma(i)}, \overline{x}_{\sigma(i)}] = [\underline{y}_{\sigma(i)}, \overline{y}_{\sigma(i)}]$ *for all* $i \in \{1, \dots, m-2\}$ *and* $[\underline{x}_{\sigma(m-1)}, \overline{x}_{\sigma(m-1)}] \prec_{B_1,B_2} [\underline{y}_{\sigma(m-1)}, \overline{y}_{\sigma(m-1)}]$
- $[\underline{x}_{\sigma(i)}, \overline{x}_{\sigma(i)}] = [\underline{y}_{\sigma(i)}, \overline{y}_{\sigma(i)}]$ *for all* $i \in \{1, \dots, m-1\}$ *and* $[\underline{x}_{\sigma(m)}, \overline{x}_{\sigma(m)}] \preceq_{B_1,B_2} [\underline{y}_{\sigma(m)}, \overline{y}_{\sigma(m)}]$

is an admissible order.

Definition 1 *Let* $A = (A_1, A_2, \dots, A_{2m})$ *be* $2m$ *aggregation functions* $A_i : [0,1]^{2m} \to [0,1]$. *The* $2m$*-tuple* A *is admissible if for all* m*-tuples of intervals* $([\underline{x}_1, \overline{x}_1], \dots, [\underline{x}_m, \overline{x}_m]), ([\underline{y}_1, \overline{y}_1], \dots, [\underline{y}_m, \overline{y}_m])$,

$$A_i(\underline{x_1}, \overline{x_1}, \dots, \underline{x_m}, \overline{x_m}) = A_i(\underline{y_1}, \overline{y_1}, \dots, \underline{y_m}, \overline{y_m})$$

for all $i \in \{1, \dots, 2m\}$ *if and only if*

$$([\underline{x}_1, \overline{x}_1], \dots, [\underline{x}_m, \overline{x}_m]) = ([\underline{y}_1, \overline{y}_1], \dots, [\underline{y}_m, \overline{y}_m]).$$

Proposition 2 *Let* A *be an admissible* $2m$*-tuple of aggregation functions. An admissible order* $\preceq_A$ *on the set of* m *intervals can be defined as*

$$([\underline{x}_1, \overline{x}_1], \dots, [\underline{x}_m, \overline{x}_m]) \prec_A ([\underline{y}_1, \overline{y}_1], \dots, [\underline{y}_m, \overline{y}_m])$$

if and only if there is a $k \in \{1, \dots, m\}$ *such that*

$$A_i(\underline{x_1}, \overline{x_1}, \dots, \underline{x_m}, \overline{x_m}) = A_i(\underline{x_1}, \overline{x_1}, \dots, \underline{x_m}, \overline{x_m})$$

for all $i \in S = \{1, \dots, k-1\}$ *and*

$$A_k(\underline{x_1}, \overline{x_1}, \dots, \underline{x_m}, \overline{x_m}) < A_k(\underline{y_1}, \overline{y_1}, \dots, \underline{y_m}, \overline{y_m}),$$

provided that $k = 1$ *induces* $S = \emptyset$.

Besides,

$$([\underline{x}_1, \overline{x}_1], \dots, [\underline{x}_m, \overline{x}_m]) = ([\underline{y}_1, \overline{y}_1], \dots, [\underline{y}_m, \overline{y}_m])$$

if and only if $\underline{x_i} = \underline{y_i}$ *and* $\overline{x_i} = \overline{y_i}$ *for all* $i \in \{1, \dots, m\}$.

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SCIENTIFIC REPORT

Improving medical decisions under incomplete data using interval–valued fuzzy aggregation

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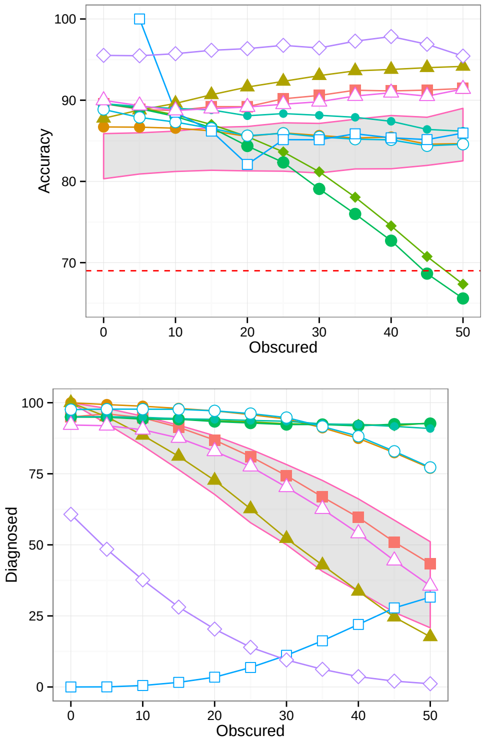
It is almost inevitable that empirical observations and real-life research produce incomplete datasets. Incomplete, or missing, data can occur for many reasons. The important point is that these reasons are natural and unavoidable, and thus the desire for complete datasets is impossible to fulfil. Clearly, missing data can have a significant effect on the conclusions that can be drawn from the data, and so it becomes a crucial issue to deal properly with missingness. In recognition of this problem, missing data analysis and decision-making under incomplete information has recently become an important area of research [1].

The present article is a contribution to the study of decision-making in the presence of incomplete information. The subject of our research is a method for supporting the medical diagnosis of ovarian tumors [2]. Since collecting all the data about a patient is often very difficult, it is crucial that the diagnostic system can give meaningful and accurate results even when some of the data is missing. We present here a novel approach that makes this possible. A key feature of our approach is that we do not use any of the known techniques for estimating missing data and data imputation, because these might significantly distort the final diagnosis. Instead, we construct a general method that makes it possible to adapt existing and well-established diagnostic methods to make them usable with incomplete data. This is achieved by interval-valued fuzzy set modelling, uncertaintification of classical methods, and finally aggregation of the incomplete results.

Our study group consisted of 268 women diagnosed and treated for ovarian tumor in the Division of Gynaecological Surgery, Poznań University of Medical Sciences, between 2005 and 2012. Among them, 62% were diagnosed with a benign tumor and 38% with a malignant tumor. The dataset is described in detail in [3].

Some results are presented in Fig. 1. Diagrams (a – accuracy) and (b – percentage of patients diagnosed) show how the aggregators and single diagnostic scales (indicated by shaded region) perform with an increasing level of missing data. The dashed horizontal line in diagram (a) indicates the accuracy of the baseline classifier.



The results presented are promising and show that the competent selection and use of aggregation methods can significantly improve the quality of decisions taken by a diagnostic system. The problem is particularly significant when the knowledge is based on incomplete information. Proper selection of the method of aggregation is essential for reducing the negative impact of data incompleteness on the quality of decisions. Because the design of an aggregation method depends on the particular problem, extensive evaluation is needed on each occasion. This can be done using our proposed method.

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SCIENTIFIC REPORT

Introducing Interpolative Boolean algebra into Intuitionistic fuzzy sets

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The main idea of this paper is to introduce Interpolative Boolean algebra (IBA) as a suitable algebra for intuitionistic fuzzy sets (IFSs). IFS theory is considered as a generalization of traditional fuzzy sets. IFSs include degrees of membership, non-membership and non-determinacy and therefore offer more descriptive power comparing to conventional fuzzy logic. IBA is a $[0, 1]$-realization of Boolean algebra, consistent with Boolean axioms and theorems. Boolean laws are secured by the uniquely mapping of logical functions to generalized Boolean polynomials. IBA is already utilized as a natural framework for consistent fuzzy logic in the sense of Boole.

In this paper, we present a Boolean consistent approach to IFSs by combining IFS with IBA. The concept of IFSs is fully retained, while IBA, with minor adaptations, is applied